HP 48G Calculator Operation and Review of Fluid Mechanics

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General Operation of Calculator

This document describes the operation of a number of subdirectories containing equations and programs to solve pipeline, open-channel, and pump flow problems. A zip file containing the directories can be downloaded at:

http://www.engineering.usu.edu/cee/faculty/gurro/MyBooks/HydPrograms.zip

You will need a cable to transfer the programs from the computer to your calculator. Information on HP's connection cable can be found at:

http://www.hp.com/calculators/accessories/connect.html

Please report any errors in this document to: gurro@cc.usu.edu

It is advisable that you store the subdirectories in a directory called HYDRAULICS to be located in the HOME directory.

Creating a directory

To create the directory HYDRAULICS use the following keystrokes:

[↦] [HOME]	Gets you into your HOME directory
[↦] [MEMORY] [NEW] [▼]	To create a new object
[α][α] [H][Y][D][R][A][U][L][I][C][S] [α] [OK]	Enter directory name
[✓ CHK]	To define object as a directory
[OK]	Shows all variables in HOME directory

You can use the [▲] and [▼] keys to move through the display to see all variables in the HOME directory.
Press [ENTER] to get to the normal display.

Transferring data with the HP48G/GX

Suppose you want to use the infrared port in the HP48G/GX to transfer a variable called MYDAT to another HP48G/GX calculator. The instructions to transfer files using the infrared port are given in page 8-2 (*Lesson 35: Transferring Objects Via Infrared*) in the *HP48 Series/Quick Start Guide*. We assume both the receiver and the sender calculators are in the appropriate directory. The steps to follow are:
Line up the infrared ports by lining up the marks located near the HP logo, just above the display. The calculators should not be farther apart than 2 inches.

Receiver calculator:

Press [↱][I/O]
Select Get from HP 48 from the menu and press [OK]

Sender calculator:

Press [↱][I/O]
Select Send to HP 48 from the menu and press [OK]
Press [*CHOOS*] and select the name of the object to be transferred (MYDAT, in this case), then press [OK]
Press [SEND]

Working with Greek letters commonly used in hydraulics

Some of the Greek letters commonly used in hydraulics are: α, β, μ, ν, σ, τ, ν(*), π, ρ, γ, ε, θ, Δ. The shortcuts for typing these characters are as follows:

α: [α][↱] [A]	β: [α][↱] [B]	μ: [α][↱] [N]	σ: [α][↱] [S]
τ: [α][↱] [T]	π: [↰][SPC]	ρ: [α][↱] [R]	γ: [α][↱] [G]
ε: [α][↱] [X] (**)	θ: [α][↱] [F]	Δ: [α][↱] [C]	

(*) The letter *nu* (ν) is not available in the HP48G/GX, thus, we will use the variable *Nu* to represent *ν*.
(**) In many hydraulics and fluid mechanics references the letter *epsilon* (ε) is used to represent the absolute roughness of a pipe or open-channel wall. Your textbook uses *e* for the absolute roughness, however, the letter *e* in the HP48G/GX is used to represent the base of the natural logarithms, namely, the number *e* = 2.718281828…. In the equations and programs described in this guidebook, we will use the variable *ee* to represent absolute roughness.

HP48G/GX Graphics: Working with function plots

While the HP48G/GX calculator provides a variety of graphic applications, one of the most useful types of graphs in hydraulics will be function plots. In this section we present an example of an HP48G/GX plot of a function of the form y = f(x). In order to proceed with the plot, first, purge the variable x, if it is defined in the current directory (because x will be the independent variable in the calculator's PLOT feature, you don't want to have it pre-defined). Press [VAR] and check if one of the white keys is labeled [*X*]. If such variable exist, purge it before proceeding.

Let's plot the Standardized Normal curve, given by,

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{z^2}{2}),$$

Define the function using the following keystrokes:

['][↰][e^x][-][α][↰][Z][y^x][2][÷][2][▶][÷][√x][↰][( )][2][×] [↰][π][ENTER]

Store it into a variable called yN:

['][α][α][↰][Y][N][α][STO]

Enter the PLOT environment, [↱][PLOT], and select `Function` as the `TYPE`.

Highlight the `EQ` field, and press [*CHOOS*]. Use the [▲] [▼] keys to select yN, then press [OK].

Type z (lowercase) as the independent variable (`INDEP`) with range -4 to 4.

Reset the independent variable in the OPTS screen: [*OPTS*][▶][4][±][OK][4][OK][OK].

Place a check mark (✓) in the `AUTOSCALE` option.

Plot the graph: [*ERASE*][*DRAW*].

To see labels: [EDIT][NXT][*LABEL*][*MENU*]

To recover the menu: [NXT][NXT][PICT]

To trace the curve with the cursor: [TRACE][(X,Y)]

Use [▶] and [◀] to move along trajectory. Check that for z = 1.05 , y = 0.231. Also, check that for z = -1.48 , y = 0.134.

To recover the menu, and return to the PLOT environment, press [NXT][CANCL].

Some useful PLOT operations for function plots

In order to discuss these PLOT options, we'll modify the function to force it to have some real roots (Since the normal curve is totally contained above the x axis, it has no real roots.) First, highlight the field in front of `EQ:` in the PLOT environment. Then, press [*EDIT*]. Use the following keystrokes:

[▼][-][.][1][ENTER].

The function to be plotted is now,

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{z^2}{2}) - 0.1.$$

Before plotting, place a check mark (✓) in the AUTOSCALE option. To plot the graph, press

[*ERASE*][*DRAW*].

Press [*FCN*] to access the calculus menu. With this menu you can obtain additional information about the plot such as intersects with the x-axis, roots, slopes of the tangent line, area under the curve, etc.

For example, to find the root on the left side of the curve, move the cursor near that point, and press [*ROOT*]. You will get the result: ROOT: -1.6635…. Press [NXT] to recover the menu.

If you move the cursor towards the right-hand side of the curve and press [*ROOT*], the result now is ROOT: 1.6635... The calculator indicated, before showing the root, that the root was found through *SIGN REVERSAL.* Press [NXT] to recover the menu.

Pressing [*ISECT*] will give you the intersection of the curve with the x-axis, which is essentially the root. Press [*ISECT*]. You will get the same message as before, namely *SIGN REVERSAL*, before getting the result I-SECT: (1.6635…, 0.0000). The [*ISECT*] function is intended to determine the intersection of any two curves closest to the location of the cursor. In this case, where only one curve, namely, f(x) as defined above, is involved, the intersection sought is that of f(x) with the x-axis. Press [NXT] to recover the menu.

Place the cursor on the curve at any point and press [*SLOPE*]. For example, if you place the cursor at any point and press [*SLOPE*], you will get the value of the slope at that point. For example, at the negative root, SLOPE: 0.16670…. Press [NXT] to recover the menu.

To determine the highest point in the curve, place the cursor near the vertex and press [*EXTR*]. The result is EXTRM: (0,0.2989…). Press [NXT] to recover the menu.

Other buttons available in the first menu are [*AREA*] to calculate the area under the curve, and [*SHADE*] to shade an area under the curve. Press [NXT] to see more options. The second menu includes one button called [*VIEW*] that flashes for a few seconds the equation plotted. Press [*VIEW*]. Alternatively, you can press the button [*NXEQ*] to see the expression for the function f(x). Press [NXT] to recover the menu.

The button [*F(X)*] gives the value of f(x) corresponding to the cursor position. Place the cursor anywhere in the curve and press [*F(X)*]. The value will be shown in the lower left corner of the display. Press [NXT] to recover the menu.

Place the cursor in any given point of the trajectory and press [*TANL*] to obtain the equation of the tangent line to the curve at that point. The equation will be displayed on the lower left corner of the display. Press [NXT] to recover the menu.

If you press [*F'*] the calculator will plot the derivative function, f'(x) = df/dx, as well as the original function, f(x). Notice that the two curves intercept at two points. Move the cursor near the left intercept point and press [*FCN*][*ISECT*], to get I-SECT: (-0.6834…,0.21585). Press [NXT] to recover the menu.

To leave the FCN environment, press [*PICT*].

Press [*CANCL*] to return to the PLOT environment.

Please notice that the field in front of EQ: in the PLOT environment now contains a list. If you press [EDIT] you will notice that there are two equations in the list:

{ '-(2*z/2*EXP(-(z^2/2))/√(2*π))' 'EXP(-(z^2/2))/√(2*π)-.1'}

When a function is chosen in the EQ: field the calculator creates a variable called EQ (for Equation) containing that function. Originally, EQ contained only f(x). After we pressed the button [F'] in the [FCN] environment, the calculator automatically added f'(x) to the list of equations in EQ.

Press [ENTER] to return to the PLOT environment.

Press [ENTER] to leave the PLOT environment.

Saving a graph for future use

If you want to save your graph to a variable, get into the PICTURE environment by pressing

[↰][PICTURE].

Then, press

[EDIT][NXT][NXT][PICT→].

This captures the current picture into a graphics object. To return to the stack, press

[PICT][CANCL].

In level 1 of the stack you will see a graphics object described as `Graphic 131 × 64`. To store it into a variable, say FIG, type

['][α][α][F][I][G][α][STO].

Your figures now is stored in variable FIG.
To display your figure again, recall it to level 1 of the stack, by pressing [FIG]. Level 1 now reads `Graphic 131 × 64`. Enter the PICTURE environment, press

[↰][PICTURE].

Clear the current picture, [EDIT][NXT][ERASE].

Move the cursor to the upper left corner of the display, by using the [◄] and [▲] keys.

To display the figure currently in level 1 of the stack press [NXT][REPL].

To return to normal calculator function, press [PICT][CANCL].

Equation solutions using the HP48G/GX calculator

Solving equations, whether a single one or a system of equations, is a major component of homework assignments in hydraulics and other engineering disciplines. In this section we present general methods available for equation solution with the HP48G/GX calculator, such as solution of systems of linear equations, solution of a single quadratic, polynomial, and non-algebraic equation, or systems of non-algebraic equations.

Systems of linear equations

System of linear equations may be generated in some hydraulics problems such as fitting data to a pump equation or using a linearized approach to the solution of pipe networks. Consider a simple system of linear equations, such as:

$$v_A + v_C = 0,$$
$$v_A - 3v_C = -32$$

This system can be re-written, using matrices and vectors, as

$$\begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} v_A \\ v_C \end{bmatrix} = \begin{bmatrix} 0 \\ -32 \end{bmatrix}.$$

Or, using matrix notation, as:

$$\mathbf{Ax} = \mathbf{b},$$

where A is the 2x2 matrix shown above,

$$x = [v_A\ v_C]^T,$$

and

$$b = [0\ -32]^T.$$

We can solve this system of linear equations by using the SOLVE environment in the HP48G or GX. For this example, use the following keystroke sequence:

[→][SOLVE][▼] [▼][▼][OK]	Select *Solve lin system*
[→][MATRIX][1][SPC][1][ENTER]	Enter matrix A
[▼][1][SPC][3][+/-][ENTER][ENTER]	
[▼][←][[ ]][0][SPC][3][2][+/-][OK]	Enter vector b
[SOLVE]	Solve for vector x. The solution is x = [-8 8].

The solutions are now displayed in level 1.

From the equation Ax = b, although not mathematically correct, we can write x = b/A (the formal expression would be x = A-1 b.) This operation can be performed directly in the HP48G or GX, as follows:

[↰][[ ]][0][SPC][3][2][+/-][ENTER]	Enter vector b
[↱][MATRIX][1][SPC][1][ENTER]	Enter matrix A
[▼][1][SPC][3][+/-][ENTER][ENTER]	
[÷]	Calculate x = b/A.

Solving quadratic equations

Quadratic equations can result from problems such as the equation describing the behavior of a centrifugal pump. For example, let the relationship between the head H and the discharge Q in a pump be given by:

$$H = 103.92\ Q - 4.903\ Q^2.$$

Suppose you are asked to determine the discharge Q required for H = 518. In other words, solve the equation:

$$103.92\ Q - 4.903\ Q^2 = 518,$$

or,

$$-4.903Q^2 + 103.92Q - 518 = 0.$$

To use the HP48G or GX calculator to solve this quadratic equation use these keystrokes:

[↱][SYMBOLIC][▲][▲][OK]	Select *Solve quad...*
[4][.][9][0][3][+/-][×][α][↰][Q][yx][2]	Type in the quadratic expression
[+][1][0][3][.][9][2][×][α][↰][Q][-][5][1][8][OK]	
[α][↰][Q]	Define the variable to solve for.
[OK]	Solves the quadratic equation symbolically.

The result shown is:

```
'Q = (-103.920+s1*25.305)/-9.806'.
```

The variable s1 should be given the values of 1 and -1 to get the two solutions. To obtain the actual solutions, use the following keystrokes:

[ENTER]	Copy the expression in the display.
[1]['][α][↰][S][1][STO]	Store the value 1 into s1.
[EVAL]	For s1 = 1, Q = 8.017.
[1][+/-][α][↰][S1]	Store the value -1 into s1.

[↰][SWAP] Swap level 1 with level 2.
[EVAL] For s1 = -1, Q = 13.178.

In the SYMBOLIC environment, when solving for a quadratic equation, you can select the principal solution, which corresponds to s1 = 1, by placing a check mark (✓) in front of the word PRINCIPAL. For this case, the principal solution is Q = 8.017.
An alternative way to solve the quadratic equation is to enter the equation into the display, then enter the solution variable, and press [↰][SYMBOLIC][QUAD]. For example, type in the expression:

'-4.903*t^2+103.92*t-518=0'.

Type the variable: 'Q'. Then, press [↰][SYMBOLIC][QUAD].

Solving polynomial equations

A polynomial equation is an equation of the form:

$$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 = 0.$$

(A quadratic equation is a polynomial equation of order 2.) The fundamental theorem of algebra indicates that there are *n* solutions to any polynomial equation of order *n*. Some of the solutions could be complex numbers, nevertheless. As an example, solve the equation:

$$3s^4 + 2s^3 - s + 1 = 0.$$

We want to place the coefficients of the equation in a vector [a_n a_{n-1}… a_1 a_0]. For this example, that vector will be [3 2 0 -1 1]. To solve for this polynomial equation using the HP48G or GX, try the following:

[↱][SOLVE][▼][▼][OK] Select Solve poly…
[↰][[ ]][3][SPC][2][SPC][0]][SPC][1][+/-][SPC][1][OK] Enter the vector of coefficients
[SOLVE] Solve for s
[ENTER] Returns to stack.
[PRG][TYPE][OBJ→][⇦][OBJ→] [⇦] All solutions will be listed in the stack.

All the solutions are complex numbers: (0.432,-0.389), (0.432,0.389), (-0.766, 0.632), (-0.766, -0.632).
Complex numbers in the HP48G or GX are represented as ordered pairs, with the first number in the pair being the real part, and the second number, the imaginary part. For example, the number (0.432,-0.389), a complex number, will be written normally as 0.432 - 0.389*i*, where *i* is the imaginary unit, i.e., *i*2 = -1.

Solving single, non-algebraic equations

This section includes the solution of almost any type of equations of one variable. For example, the cross-sectional area in a circular pipe functioning as an open channel is given by the expression:

To solve for β given D = 2.5 and A = 2.2, we will first set the angular units to radians by using the keystroke combination: [↰][RAD], until the indicator RAD is shown in the upper left corner of the display. Then, use the following to solve for β:

[↱][SOLVE] [OK] Select `Solve equation`
['][α][A][↰][=][↰][( )][α][D][yx][2][÷][4][▶][×][↰][( )]
[α][↱][B][-][SIN] [α][↱][B] [▶][×][COS] [α][↱][B]
[ENTER] Type in the equation

As you define the equation in the EQ field the calculator creates fields for the variables involved, namely A, D and β. Enter the values of the known variables,

Enter the following values:

[2][.][2][OK] [2][.][5][OK]

The β field is now highlighted, to solve for β just press [SOLVE]. The result is 1.4892…

Note: For equations where there is no single solution, you can type in an initial guess for the unknown in the appropriate field before hitting the [SOLVE] key. As an example, try solving the quadratic equation shown above using the SOLVE environment.

When done press [ON] or [ENTER]. This will return you to the normal calculator display, which will show the solution you just found. You will also notice that there will be variables created to hold the values of A, D, β, and the equation EQ. The variable EQ, which can be defined locally in a sub-directory without affecting its value in other sub-directories, is a reserved variable that the calculator uses to store equations or systems of equations for the SOLVE or PLOT environments.

Solving systems of non-algebraic equations through an iterative process

This approach will be illustrated later in the guidebook when solving the problem of a pipeline connecting two reservoirs (subdirectory TWRS within PIPES). The equations constituting the system are stored in the variable EQ as a list of equations.

Solving systems of non-algebraic equations using [↰][SOLVE][ROOT][SOLVR]

This approach will be illustrated later in the guidebook when solving the problem of a pipe functioning as an open-channel using Manning's equation (subdirectory MANC within CHANNELS).

Solving systems of equations using the SOLVESYS library

The SOLVESYS library is a binary library developed by a Finnish computer scientist by the name of Sune Bredahl. He has made this library available for free through the internet. Information on the use of the library can be found in the URL:

http://www.gbar.dtu.dk/~c947086/hp48.html

This library utilizes a generalized Newton-Raphson method for the solution of a system of algebraic or non-algebraic equations. The system of equations needs to be stored as a list in the variable EQ. Once activated, SOLVESYS lets the user initialize constants and unknowns and tries to find a solution to the system.

This approach is used in several of the solutions presented in this guidebook.

REVIEW OF FLUID MECHANICS

Systems of Units

We use two systems of units, the *Systeme International (SI)*, the modern version of the metric system, and the English System (ES). Systems of units are defined by a few basic units upon which all other units are built.
In the SI the basic units for applications in fluid mechanics are those of length, the *meter* (m), of time, the *second* (s), and of mass, the *kilogram* (kg). From them, for example, we can define other units such as:

- Force, *Newton* = kg · m/s2
- Energy, *Joule* = Newton · m
- Power, *Watt* = Joule/s
- Pressure, *Pascal* = Newton/m2

Other standard units of length in the SI are the *millimeter* (mm = 10-3 m) and the *kilometer* (km = 103 m). In many practical applications, particularly in chemical and environmental engineering, the *gram* (g = 10-3 kg) is used as a standard unit of mass. Commonly, temperature in the SI is measured in *centigrade degrees* (oC) defined such that the freezing point of pure water is at 0oC and the boiling point of the same liquid is at 100oC. Absolute temperature in the SI is measured in *Kelvin* (K) defined as

$$T(K) = t(^oC) + 273.15,$$

where T is absolute temperature and t is the temperature in centigrade degrees.
Prefixes are used to indicate multiples and fractions of the principal units. Among the most commonly used are:

- Mega (M) = 106
- Kilo (K) = 103
- milli (m) = 10-3
- micro (μ) = 10-6

The *centimeter* (cm), although not a standard SI unit, is still commonly used in measurement. Constants commonly used in the SI system in hydraulic applications are the acceleration of gravity, g = 9.806 m/s2, and the density of water at 4oC, ρw = 1000 kg/m3.

In the ES the basic units are those of length, the *foot* (ft), of time, the *second* (s), and of force, the *pound* (lb). Some derived units in the ES are:

- Mass, *slug* = lb · s2/ft
- Power, *horsepower* (HP) = 550 lb·ft/s

The *inch* (in = 1/12 ft) and the *mile* (mi = 5280 ft) are commonly used for measuring length in the ES. The *Kip* (Kip = 1000 lb) is commonly used in engineering for measuring forces. The *pounds-per-square-inch* (psi = 144 lb/ft^2) are used for measuring pressure. Temperature in the ES is measured in *Fahrenheit degrees* (^oF) with the freezing and boiling points of water corresponding to 32^oF and 212^oF, respectively. The corresponding absolute temperature is measured in *Rankine* (R) defined as

$$T(R) = t(^oF) + 459.67,$$

where T is absolute temperature and t is the temperature in Fahrenheit degrees.

Important constants in the ES system are the acceleration of gravity, g = 32.2 ft/s2, and the specific weight of water at 4^oC, γ_w = 62.4 lb/ft3.

Both the SI and ES have the same units of time, therefore, when necessary, besides the basic unit, the second (s), other units used are:

- *Minute*, min = 60 s
- *Hour*, hr = 60 min
- *Day*, dy = 24 hr
- *Year*, yr = 365 dy

Some basic conversion factors between the two systems of units are the following:

- 1 in = 2.54 cm = 25.4 mm
- 1 m = 3.28 ft
- 1 ft = 0.3048 m
- 1 mi = 1.609 km
- 1 kg = 2.2 lb

Temperature conversions are given by the formulas:

- $t(^oC)/(t(^oF) - 32) = 5/9$
- $T(K) = t(^oC) + 273.15$
- $T(R) = t(^oF) + 495.67$
- $T(K)/T(R) = 5/9$

T represents absolute temperature (K or R) while t represents temperature in centigrade or Fahrenheit degrees.

Note: while the HP48G calculator allows you to work with units, care must me exercised to use the proper unit definition as the units in the calculator do not necessarily follow the rules of the SI or ES systems.

Derivation of the units of measurement is not difficult if we understand well the definition of the quantity of interest. The following are simple definitions of many quantities commonly utilized in solid and fluid mechanics. Use them, if necessary, to verify or derive units: Given L = length, T = time, M = mass, F = force, g = acceleration of gravity, ρ_w = density of water at 4^oC, γ_w = specific weight of water at 4oC,

- Area, $A = L^2$
- Volume, $Vol = L^3$
- Velocity, $V = L/T$
- Acceleration, $a = L/T^2$
- Volumetric flow rate or discharge, $Q = Vol/T$
- Mass flow rate, M(dot) = M/T
- Density, $\rho = M/Vol$
- Force, $F = M \cdot a$
- Weight, $W = M \cdot g$
- Specific weight, $\gamma = W/Vol$
- Pressure or stress, $P = F/A$
- Work or Energy, $E = F \cdot L$
- Power, $Power = E/T = F \cdot V$

Fluid Properties

Fluid properties of interest in hydraulic applications are the following:

- Fluid density, $\rho = m/Vol$, mass per unit volume [kg/m3, slug/ft3].

- Specific weight, $\gamma = W/Vol$, mass per unit volume [N/m3, lb/ft3].

- Specific density, $S = \rho/\rho_w = \gamma/\gamma_w$, ratio of a fluid density or specific weight to the corresponding quantity for water at 4oC [dimensionless].

Note: water has the curious ability of reducing its density with temperature until $t = 4^oC$, and then increasing it as the 0^oC point is approached. Since 0oC is the freezing point of water, ice is slightly less dense than water and for that reason it floats. The density of water at its minimum value, namely at 4^oC, is therefore, used as a reference point in defining specific gravity.

- Absolute or Dynamic Viscosity, $\mu = \tau/(dv/dy)$, ratio between the shear stress at a point and the local velocity gradient [Pa·s = kg/m·s].

- Kinematic viscosity, $\nu = \mu/\rho$, ratio of the dynamic viscosity to the fluid density [m^2/s, ft^2/s].

- Bulk modulus of elasticity, $E = \Delta p/(\Delta Vol/Vol)$, ratio of pressure increase to relative volume decrease [Pa, lb/ft^2 or psi].

Note: Typically, hydraulic applications consist of the flow of liquids, particularly water. Unlike gases, liquids possess large bulk modulus of elasticity meaning that a very large pressure increment is required in order to show any appreciable change in volume. Such fluids are referred to as *incompressible fluids*.

- Surface tension, σ, force per unit length normal to a liquid-vapor interface due to molecular attraction in the surface [N/m, lb/ft].

- Vapor pressure, p_v, pressure in the vapor phase near a liquid-vapor interface when the rate of evaporation is in equilibrium with the rate of condensation through the surface [Pa, lb/ft2 or psi].

HP 48/49 Calculator Application: Subdirectory WATERPROP (WATE)

Programs within this sub-directory can be used to obtain physical properties of water. The properties provided are the density, kinematic viscosity, and vapor pressure of water at different temperatures. To obtain properties in the International System of units enter the temperature in ^oC and press [S.I.]. For properties in the English System of units enter the temperature in ^oF and press [E.S.].

Examples:

25 [S.I.] shows: *T(oC): 25; ρ (kg/m^3):997; Nu(m^2/s): 0.000000893; and, pv(kPa-abs): 3.17.*

65 [E.S.] shows: *T(oF): 65; ρ(slug/ft^3): 1.937; Nu(m^2/s): 0.00001138; and, pv(psi-abs): 0.31* .

The subdirectory includes also the programs [C→F] and [F→C] that can be used to convert temperature from centigrade to Fahrenheit degrees and vice versa. To use these programs simply enter the value of the temperature that you want to convert (omit the units) and press the appropriate softkey.

The [↑UP] button can be used to get to the upper directory.

This sub-directory is also available as a HP 48/49 library

Hydrostatics

Hydrostatics is the study of fluids in equilibrium. In particular we are interested in determining the variation of the pressure with position. The *basic equation of hydrostatics* indicates that the pressure in a fluid at any depth h is given by

$$p = p_o + \gamma \cdot h,$$

where p_o is a reference pressure at h = 0, and γ = specific weight of the fluid.

Force on a plane area

Integrating the pressure distribution over a plane area produces the total normal *force over the area* as

$$F = p_c \cdot A = \gamma \cdot h_c \cdot A,$$

where pc is the pressure at the geometric center (or centroid) of the area.

Absolute and Gage Pressure

Absolute pressure is measured by using a perfect vacuum, i.e., the total absence of matter, as the point of zero pressure. Thus, absolute pressure is always a positive quantity. *Gage pressure*, on the other hand, utilizes the atmospheric pressure as the point of zero pressure. In other words, gage pressure actually measures the difference between the pressure of interest and the atmospheric pressure. Gage pressure, therefore, can be positive or negative. The relationship between the two pressure scales can be written as

$$p_{abs} = p_{gage} + p_{atm},$$

where p_{abs} is the absolute pressure, p_{gage} is the gage pressure, and p_{atm} is the atmospheric pressure.

The *atmospheric pressure*, measured in the absolute pressure scale, is equal to 101.325 kPa or 2116.21 lb/ft^2. In the ES this value is typically given in pounds per square inch (psi) as 14.6959 psi.

Because atmospheric pressure is measured with an instrument called barometer, the atmospheric pressure is also referred to as the *barometric pressure*. The simplest barometer consists of a long glass tube closed on one end and filled with mercury (S = 13.56). The tube is filled completely with mercury and its open end is carefully submerged in a container also filled with mercury. The container is open to the atmosphere. The weight of the mercury column in the tube will be balanced by the weight of the atmosphere on the free surface of the open container. If the glass tube is long enough (say, about 1 m long), the mercury column within it will show a vacuum on top once the tube is inverted and its opening submerged in mercury. Measurements indicate that the length of the mercury tube corresponding to the standard atmospheric pressure is 760 mm. Thus, sometimes you will see the atmospheric pressured given as 760 mm-Hg or 29.9213 in-Hg.

Water (S = 1.0) could be use as a barometric fluid instead of mercury, however, the length of the water column required to balance the weight of the atmosphere turns out to be 10.33 m or 33.90 ft. The length of the tube required to construct the barometer makes the construction of a water barometer practically impossible. Nevertheless, the atmospheric pressure is sometimes reported as 10.33 m-H_20 or 33.90 ft-H_20. In meteorological applications the atmospheric pressure is sometimes given in bars (1 bar = 100 kPa = 2088.54 lb/ft2 = 14.5038 psi) as 1.01325 bars, or simple as 1 atmosphere.

Manometers

The laws of hydrostatic are also utilized to measure pressures through the use of *manometers*. A manometer is a tube containing one or more immiscible fluids. The surface of separation between immiscible fluids is known as the meniscus. To write the manometer's equation we mentally "travel" through the length of the manometer starting at a point of known pressure, say point A. Follow these steps:

- Write down the pressure for point A, say p_A, as the starting term of the equation.
- Mentally "travel" through the manometer towards the next meniscus. If the meniscus is located above point A add to p_A the contribution of the fluid weight, namely, the specific weight of the fluid multiplied by the vertical distance between point A and the meniscus. If the meniscus is located below point A subtract that contribution.
- Continue "travelling" mentally through the manometer towards the next meniscus, adding the fluid weight contribution if the next meniscus is located above the current meniscus or subtracting those contributions in the opposite situation.
- Repeat step 3 until reaching the other end of the manometer, say, point B.
- Make the equation you wrote in steps 1 through 3 equal to the pressure at point B, say p_B.

Unless the pressure at A, or B, is known, you will need to solve for a pressure difference, either, $p_A - p_B$, or $p_B - p_A$. Gases have negligible specific weights compared to liquids, therefore, the pressure in a gas is considered constant throughout the entire gas volume.

Fluid Dynamics

Fluid dynamics describes the motion of fluids. According to whether the flow properties change with position (x, y, z), or time (t), a flow can be classified as uniform or non-uniform, or as steady or unsteady. Let ϕ be any flow property, e.g., velocity, temperature, density, etc., the flow is *steady* if $\partial\phi/\partial t = 0$, otherwise, it is *unsteady*. In this class we will deal mainly with *one-dimensional* flow, meaning, that the flow properties are assumed to vary only along the main flow direction. Let s be the position along that direction, then, a flow is *uniform* is $\partial\phi/\partial s = 0$, else it is *non-uniform*.

Laminar and Turbulent Flows

Experiments performed by Osborne Reynolds in England in the 19th century revealed that at relatively low flow velocities in a fluid the fluid particles tend to move in layers (Latin *laminae*). This regular, well-organized flow pattern is referred to as *laminar flow*. Molecular exchange between fluid particles in laminar flow manifest itself in the larger scale as normal stresses (pressures), and shear stresses between the layers. These stresses are referred to as laminar or viscous stresses.

As the velocity increases the flow layers become unstable, the layers break up and the flow particles organize themselves into structures known as *turbulent eddies*. At this point the flow has become *turbulent* and eddies of a wide range of sizes are observed. The exchange of momentum between the different sizes of turbulent eddies produce shear stresses of orders of magnitude much larger than those observed in laminar flows. These stresses are known as Reynolds or turbulent stresses. Turbulent flow, due to its chaotic nature, is more efficient in achieving mixing of any contaminant present in the flow.

The One-dimensional Approach

The idea behind the assumption of one-dimensional flow is that we describe average properties of the flow as they vary along the main flow direction. For example, in laminar flow in a pipe we know that the local velocity at any point of the cross-section is not uniform across the area. This distribution is described by a parabolic velocity distribution given by

where r is the distance from the pipe centerline, and R is the radius of the pipe. Obviously, the local flow velocity, while being constant along a circle r = constant varies from the pipe centerline, where v(0) = vmax, to the pipe wall, where v(R) = 0.
To simplify the analysis we deal with the average or mean velocity, V, defined as:

The expression shown above applies to any flow in a pipe where the local velocity is a function of the radial distance r only. If we let s indicate the position of a given cross-section of the flow along the main flow direction, then V will be, in general, a function of s, i.e.,

V = V(s).

Other properties, e.g., density, temperature, can also be analyzed by reducing them to an average quantity, as done with the velocity above, which is a function of s only.

Dimensionless Numbers

Dimensionless numbers are developed in fluid mechanics based on the ratio of relevant forces and are used to ensure similarity between laboratory models and full-size prototypes. The dimensionless numbers are defined in terms of certain average quantities of the flow such as:

- A characteristic length, L. This could be, for example, the diameter of a pipe, the length of a boat, the diameter of a sphere, etc.
- A characteristic velocity, V. For example, the average velocity in a pipe, the uniform velocity around a submerged body, etc.

The dimensionless numbers also include the following fluid properties in their definitions:

- Density, ρ.
- Dynamic viscosity, μ, or kinematic viscosity, $\nu = \mu/\rho$.

- Bulk modulus of elasticity, E.
- Surface tension, σ.

Finally, acceleration of gravity, g, is also involved.

The most important type of forces acting on fluid flows are listed below. Their magnitude is shown herein as being proportional to combinations of the variables listed above:

- *Inertial forces*: basically the *mass* × *acceleration* terms in Newton's second law applied to fluids ~ ρ·V2·L2
- *Pressure forces*: present in all fluid flows ~ Δp·L2.
- *Viscous forces*: due to the effects of viscosity, typically shear forces ~ μ·V·L
- *Gravity forces*: due to the weight of the fluid ~ ρ·g·L3
- *Elastic forces*: represent the effects of compressibility ~ E·L2
- *Surface tension forces*: manifest in the interface between liquids and gases ~ σ·L

Based on the ratio of these forces we can form the following dimensionless numbers:

- *Euler number*, Eu = pressure forces/inertial forces = Δp/ρ·V2. This is basically a pressure coefficient.
- *Reynolds number*, Re = inertial forces/viscous forces = ρ·V·L/μ = V·L/ν.
- *Froude number*, Fr = (inertial forces/gravity forces)1/2 = V/(g L)1/2.
- *Mach number*, Ma = (inertial forces/elastic forces)1/2 = (ρ·V/E)1/2 = [V/(E/ ρ)]1/2 = [V/c]1/2. c = speed of sound in the fluid.
- *Weber number*, We = inertial forces/surface tension forces = ρ·V2·L/σ.

In most of the hydraulic applications presented in this class we deal mainly with the Reynolds number (for pipeline and hydraulic machinery applications) and the Froude number (for open channel applications).

Conservation Equations

The application of the laws of conservation of mass, momentum, and energy to fluid flows produces the so-called continuity, momentum, and energy equations. The expressions presented below correspond to one-dimensional flows with cross-sections perpendicular to the main flow direction. Such are the cases of flow in pipelines and in open channel flow.

Continuity

A version of the continuity equation was presented earlier when defining the average velocity in a cross-section of a pipe. The most general form of the continuity equation for an incompressible fluid in a single conduit is given by

$$Q = V_1 \cdot A_1 = V_2 \cdot A_2 = V_3 \cdot A_3 = \ldots$$

where the sub-indices describe different flow cross-sections. The area of each cross-section is represented by A while V is the average velocity in the cross-section. The quantity Q is the volumetric flow rate or discharge.

For a conduit that splits into a number of connected conduits the following form of the continuity equation applies:

$$Q = Q_1 + Q_2 + Q_3 + \dots,$$

or,

$$V \cdot A = V_1 \cdot A_1 + V_2 \cdot A_2 + V_3 \cdot A_3 + \dots$$

If the velocity distribution in a cross-section is given, it will typically be described as $v = v(x,y)$, or $v = v(r,\theta)$. In such case, the continuity equation can be written as:

Momentum

The momentum equation for fluid flow requires the selection of a control volume and the analysis of forces on the volume as well as the momentum fluxes through the control surface. Typically, for one-dimensional flow, the control volume is selected so that it is limited by an upstream and a downstream cross-section. Care must also be exercised to include forces that can be easily obtained. Forces typically included are the weight of the fluid in the control volume, forces applied by external elements on the control volume, normal forces on the limiting cross-sections (hydrostatic forcers), and shear forces on the control surface.

The flux of momentum per unit area across any given cross-section is the product $\rho \cdot Q \cdot V$. Because the local velocities in a cross-section are not uniform, the momentum flux needs to be corrected by multiplying a *momentum correction coefficient*, also known as Boussinesq coefficient, β, so that it now reads $\beta \cdot \rho \cdot Q \cdot V$. The momentum correction coefficient is defined as

$$\beta = \frac{1}{A}\int_A \left(\frac{v}{V}\right)^2 dA.$$

Since momentum is a vector quantity, momentum equations can be written, in general, for each of the coordinate directions, namely,

$$\Sigma Fx = \Delta(\beta\rho QV)_x,$$
$$\Sigma Fy = \Delta(\beta\rho QV)_y,$$
$$\Sigma Fz = \Delta(\beta\rho QV)_z.$$

Where ΣF represents the sum of forces on the control volume along the direction of interest, and the operator Δ implies the net increase in the corresponding momentum flux, i.e., momentum flux out minus momentum flux in. Care must be exercised that these momentum fluxes have the proper sign depending of the direction of the flow. Problems involving changes of direction in pipelines and the moving vanes of a centrifugal pump or radial turbine utilize these forms of the momentum equations.

In the case that the flow is mainly in the x direction, and labeling the upstream (inflow) cross-section as 1 and the downstream (outflow) cross-section as 2, we can write a single momentum equation as:

$$\Sigma Fx = (\beta\rho QV)_2 - (\beta\rho QV)_1.$$

This will be the case for many an application in open-channel flow.

Energy

Let H represent the total mechanical energy per unit weight available at any given cross-section. H has units of length, and it is often referred to as the *total energy head*. It can be shown that H is made up three main components:

Gravitational potential energy per unit weight = mgz /mg = z, i.e., the *elevation* of the section above an arbitrary reference level or datum.

Mechanical energy due to pressure per unit weight = p·Vol / γ·Vol = p/γ, a *pressure head*.

Kinetic energy per unit weight = $(1/2)mV^2$ / mg = $V^2/(2g) = h_v$, a *velocity head*.

As in the case of the momentum flux, the kinetic energy per unit weight at a given cross-section needs to be modified by a *kinetic energy correction coefficient*, α. This coefficient accounts for the local velocity being non-uniform across a given flow cross-section. The coefficient α is obtained from

$$\alpha = \frac{1}{A}\int_A \left(\frac{v}{V}\right)^3 dA.$$

The velocity head, or kinetic energy per unit weight, is re-defined as

$$h_v = \alpha(V^2/2g),$$

and the total head at the cross-section is now

$$H = z + p/\gamma + \alpha(V^2/2g).$$

The combination (z + p/ γ) is known as the *piezometric head* because it represents the elevation of the meniscus in a piezometer connected to the point of interest.

The energy equation that is commonly utilized for solving hydraulic problems is actually the equation of conservation of mechanical energy between two cross-sections of a fluid flow. Let section 1 represent the upstream cross-section of interest, and section 2 the downstream cross-section in the energy equation. Let H1and H2 represent the total energy heads at section 1 and 2, respectively. In an *ideal fluid* flow, where viscous forces are negligible, there will be no energy losses, therefore, we can write,

$$H_1 = H_2,$$

or,

$$z_1 + p_1/\gamma + \alpha(V_1^2/2g) = z_2 + p_2/\gamma + \alpha(V_2^2/2g).$$

This version of the energy equation for ideal fluid flows is known as the *Bernoulli equation*. The equation is commonly applied to situations were the energy losses are negligible such as flow meters in pipe flow and transitions in open channel flows.

The flow of *real fluids* always include energy losses due mainly to friction. The presence of structures or devices in the flow path can also produce localized energy losses known as minor losses (e.g., elbows or valves in a pipeline). Flow energy can also be removed by a turbine, or added to the flow by a pump. To include these energy inputs and outputs in the general energy equation we will define the following terms:

- Energy losses due to friction per unit weight, hf.
- Minor losses per unit weight, hL.
- Head removed by a turbine, HT.
- Head added by a pump, HP.

The most general form of the energy equation for a real fluid flow including pumps and turbines is therefore

$$H_1 + H_P = H_2 + H_T + h_f + h_L,$$

or,

$$z_1 + p_1/\gamma + \alpha_1(V_1^2/2g) + H_P = z_2 + p_2/\gamma + \alpha_2(V_2^2/2g) + H_T + h_f + h_L.$$

While this equation is appropriate for most *close-conduit flows*, in *open-channel flow* the pressure head, p/γ, is typically replaced by the flow depth, Y. Pumps, turbines, and minor losses are not usually accounted for in open-channel flows. In such cases, the energy equation will simplify to

$$H_1 = H_2 + h_f,$$

or,

$$z_1 + Y_1 + \alpha_1(V_1^2/2g) = z_2 + Y_2 + \alpha_2(V_2^2/2g) + hf .$$

Momentum and Energy Equations in Turbulent Flows

Measurements of velocity distributions in turbulent flows indicate that the velocity tends to be more uniform across the cross-section that in laminar flows. The practical implication of this fact is that the values of the momentum correction coefficient, β, and the energy correction coefficient, α, for turbulent flows are more often than not very close to 1.0. Unless indicated otherwise, most of the applications of the momentum and energy equation in hydraulics will utilize values of β and α equal to 1.0. The momentum and energy equations will simplify, therefore, the following expressions for most turbulent flows:

- General form of momentum equations in the three coordinate directions:

$$\Sigma Fx = \Delta(\rho QV)_x,$$
$$\Sigma Fy = \Delta(\rho QV)_y,$$
$$\Sigma Fz = \Delta(\rho QV)_z.$$

- Momentum equation for flows mainly in the x-direction, inflow cross-section is 1, outflow cross-section is 2:

$$\Sigma Fx = (\rho QV)_2 - (\rho QV)_1.$$

- Bernoulli equation, applied whenever energy losses are negligible:

$$z_1 + p_1/\gamma + V_1^2/2g = z_2 + p_2/\gamma + V_2^2/2g.$$

- General form of the energy equation in close conduits including friction losses (hf), minor losses (hL), head extracted by a turbine (HT), and head added by a pump (HP):

$$z_1 + p_1/\gamma + V_1^2/2g + HP = z_2 + p_2/\gamma + V_2^2/2g + H_T + h_f + h_L.$$

- General form of the energy equation in open channel flows:

$$z_1 + Y_1 + \alpha_1(V_1^2/2g) = z_2 + Y_2 + \alpha_2(V_2^2/2g) + h_f\ .$$

